

We are given some pairing function $e : G_1 \times G_2 \rightarrow G_T$, elements $g_1 \in G_1, g_2 \in G_2, |G_1| = |G_2| = |G_T| = p$, and some hash $H : \mathcal{M} \rightarrow G_1$. The code calculates some Lagrange basis polynomials and evaluates them at $x = 0$. Let's call these values l_0, l_1, L_0, L_1, L_2 to be exact. We input the values $H(m)^a, H(m)^b, g_2^{x_0}, g_2^{x_1}, g_2^{x_2}$ and are constrained by $g_2^{x_0}$ and $g_2^{x_1}$ should be one of the existing public keys. Ideally m should be "this stuff", so we can precompute $H(m)$. To validate the pairing, the code verifies

$$e(H(m)^{al_0} \cdot H(m)^{bl_1}, g_2) = e(H(m), g_2^{x_0L_0} \cdot g_2^{x_1L_1} \cdot g_2^{x_2L_2}).$$

We simplify this to

$$e(H(m), g_2)^{al_0+bl_1} = e(H(m)^{al_0+bl_1}, g_2) = e(H(m), g_2^{x_0L_0+x_1L_1+x_2L_2}) = e(H(m), g_2)^{x_0L_0+x_1L_1+x_2L_2}.$$

Therefore the exponents must be equal. We can then simplify to

$$al_0 + bl_1 = x_0L_0 + x_1L_1 + x_2L_2.$$

Since x_2 is totally unconstrained, se just solve for it.

$$x_2 = \frac{1}{L_2} (al_0 + bl_1 - x_0L_0 - x_1L_1).$$

In the end, we need to operate on the public keys $g_2^{x_0}$ and $g_2^{x_1}$ so we lift this back into G_2 and use some math

$$g_2^{\frac{1}{L_2}(al_0+bl_1-x_0L_0-x_1L_1)} = \left(g_2^{al_0+bl_1} \cdot (g_2^{x_0})^{-L_0} \cdot (g_2^{x_1})^{-L_1} \right)^{\frac{1}{L_2}}.$$

Now the RHS is something we can compute given choices for a and b . Problem solved.