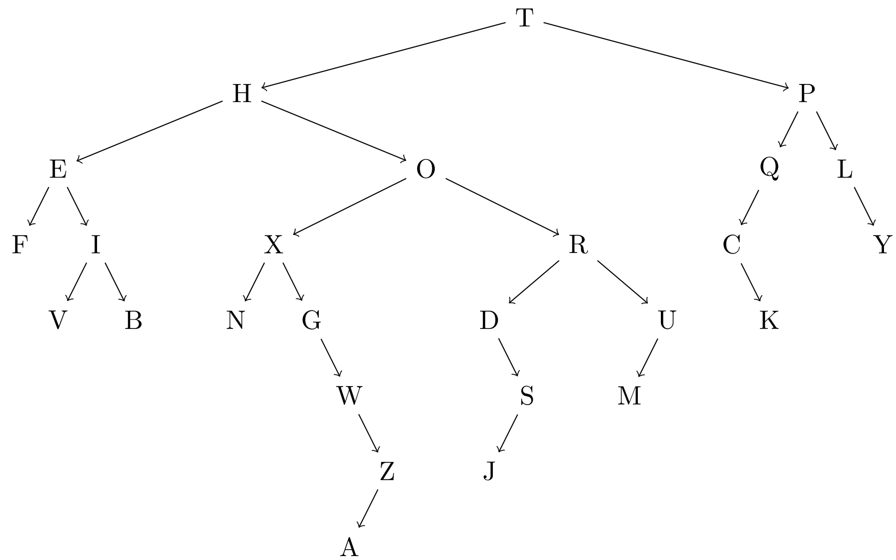
1. (a) THEFIVBOXNGWZARDSJUMPQCKLY

   (b)



2. (a) We proceed via strong induction on $|w|$. In particular, we have the following cases.

   $|w| = 0$. Then $w = \varepsilon$ and $|\operatorname{swap}(\varepsilon)| = |\varepsilon|$.

   $|w| = 1$. Then $w = 0$ or $w = 1$. For the case $w = 0$, we have $|\operatorname{swap}(0)| = |0|$. The case $w = 1$ proceeds analogously.

   Now assume that $|\operatorname{swap}(u)| = |u|$ for all $|u| < |w| = k$, for some positive integer $k \geq 2$. We show that the proposition holds for w. In particular, we can write $w = abx$, where a and b are letters and x is a string, and

   $$|\operatorname{swap}(abx)| = |ba \cdot \operatorname{swap}(x)| = 2 + |\operatorname{swap}(x)| = 2 + |x| = |abx|,$$

   where $|\operatorname{swap}(x)| = |x|$ follows from our induction hypothesis.

   (b) Again we proceed via strong induction on $|w|$. We have the following cases.

   $|w| = 0$. Then $w = \varepsilon$ and $\operatorname{swap}(\operatorname{swap}(\varepsilon)) = \varepsilon$.

   $|w| = 1$. Then $w = 0$ or $w = 1$. For the case $w = 0$, we have $\operatorname{swap}(\operatorname{swap}(0)) = 0$. The case $w = 1$ proceeds analogously.

   Now assume that $\operatorname{swap}(\operatorname{swap}(u)) = u$ for all $|u| < |w| = k$ for some positive integer $k \geq 2$. We show that the proposition holds for w. In particular, we (again) write $w = abx$, where a and b are letters and x is a string, and compute

   $$\operatorname{swap}(\operatorname{swap}(abx)) = \operatorname{swap}(ba \cdot \operatorname{swap}(x)) = ab \cdot \operatorname{swap}(\operatorname{swap}(x)) = abx,$$

   as desired, where the final equality follows from the induction hypothesis.

3. (a) This is clear by part (c), because $\#(1, 101110101101011) = 10$.

   (b) We proceed via strong induction on $|w|$.

   The case $|w| = 0$ is trivial as $\#(1, \varepsilon) = 0$.

   Now assume that if $|u| < |w| = k$ for some positive integer k and $u \in L$, then u has an even number of 1's. We show that if $w \in L$ then w must also have an even number of ones. In particular, because $|w| > 0$, we have either $w = 0x$ or $w = 1x1y$ for some strings $x, y \in L$. In the case $w = 0x$, we have $\#(1, 0x) = \#(1, x)$, where the latter is even by the induction hypothesis. In the case $w = 1x1y$, then $\#(1, 1x1y) = 2 + \#(1, x) + \#(1, y)$, but the last two summands are even by the induction hypothesis (because $x, y \in L$) so their sum must also be even, as desired.

   (c) We proceed via strong induction on the number of ones, restricted to even non-negative integers.

   If $\#(1, w) = 0$, then w is constructed entirely of zeroes and the strings $\varepsilon, 0, 00, 000, \ldots$ (any element of $\{0\}^*$) are in L via the construction $x \in L \implies 0x \in L$.

   Now assume that if $\#(1, u) < \#(1, w) = 2k$ for some positive integer k and $\#(1, u)$ is even, then $u \in L$. We will show that $w \in L$ as well. There are two cases. Either $w = 0x$ or $w = 1x$. It suffices to prove the case $w = 1x$ because we can prepend any number of zeroes via $x \in L \implies 0x \in L$. Because $\#(1, w) = 2k > 0$, we can write $w = 1x1y$ for some strings x, y, where the length of y is minimal. This means that y is a member of the set $\{0\}^*$. Then $\#(1, x) = 2k - 2 < 2k \implies x, y \in L$ by the induction hypothesis so $1x1y \in L$ as desired.